PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 2 JANUARY 15, 1916 Number 1

A POSSIBLE ORIGIN FOR SOME SPIRAL NEBULAE

By George F. Becker

UNITED STATES GEOLOGICAL SURVEY, WASHINGTON

Read before the Academy, November 17, 1915. Received, November 22, 1915

In speculations on the evolution of nebulae it has become the fashion to postulate an initial spheroid consisting exclusively of elastic fluids, this assumption lending itself most readily to exact reasoning and computation.

Kant, Herschel, and Laplace, however, did not assume gaseous nebulae. In a paper on Kant as a natural philosopher, printed in 1898, I gave an outline of his hypothesis including the following passage:[1]

Tendencies to motions in all directions, excepting in one resultant plane, are suppressed by mutual interferences of the free particles. Most of the material accumulates at the center, in the sun, but a wide, thin disc of heterogeneous matter remains. This disc consists of discrete particles each of which has acquired such a velocity and direction as to maintain the appropriate orbital motion. . . . . Mutual attraction and adhesion, beginning at relatively massive particles, cause the agglomeration of the particles in any zone or ring to single planets or to groups of planetary bodies.

From this and other passages it appears that to this very original thinker solid particles were the most essential components of nebulae.

William Herschel's nebular hypothesis was founded upon induction and observations. Nowhere in his own works do I find so graphic a résumé as the following which Laplace gave in the *Système du Monde*:[2]

Herschel, while observing the nebulae by means of his powerful telescopes has followed the progress of their condensation, not in a single one, for this progress could not become sensible to us until centuries had passed, but in the aggregate; as in a vast forest one traces the growth of the trees among the individuals of diverse ages which it contains. He first observed nebulous

matter spread out in diverse bodies in different parts of the heavens of which it occupies a great area. He saw in some of these bodies the nebulous matter slightly condensed about one or several slightly luminous nuclei. In other nebulae, these nuclei shone more brightly relatively to the nebulosity which surrounded them. The atmosphere of each nucleus was about to segregate by a final condensation: multiple nebulae resulted formed of brilliant nuclei very close together and each surrounded by an atmosphere: occasionally the nebulous matter condensing uniformly produced the nebulae called *planetary*. Finally a greater degree of condensation transformed all of these nebulae into stars. The nebulae classified according to this philosophic view indicate with extreme probability their future transformation into stars and the former nebulous state of the stars now in existence.

It will be observed that nebulae with a single nucleus are mentioned only as special cases of multinuclear and, therefore, heterogeneous nebulae. Herschel himself made repeated references to the character of nebulous matter. For him it was anything which could shine, and while he was unable to come to definite conclusions, he regarded it as probable that it comprised matter in all three states.

Laplace's information as to nebulae was derived almost wholly from Herschel; so far as I can ascertain he made no telescopic observations of his own on these bodies, and his ideas of the constitution of nebulae seem to have been identical with those of the great English observer. Both in the passage quoted above and in his famous note on the nebular hypothesis Laplace referred to atmospheres, but they were cloudy or dusty atmospheres not inconsistent with the presence of multiple nuclei; they were characterized by a 'numberless variety of densities' and seem to have resembled volcanic clouds. In other essays, such as that on the barometer, Laplace showed his complete mastery of Boyle's and Gay Lussac's laws and in these memoirs he frequently employs the word gas. In his note on the nebular hypothesis he never once uses this word and the very first application he made of his theory was to offer an explanation of the genesis of the Pleiades. In his time, the presence of nebulous matter in this cluster was unknown, but the photographic plate shows that it is embedded in one of the most astonishing of the great nebular masses. Laplace's selection of the Pleiades was thus a striking example of his prescience.

Much as more modern science has contributed to knowledge of the nebulae, the question of their constitution is still unsettled. That some of them contain luminous gas (helium, hydrogen, nebulium) was shown by Huggins 50 years ago, but many nebulae and among them the spirals show continuous spectra. The continuity might be due to

gases at high pressures; but that gases should assume so complex a configuration as that of a spiral is difficult to understand while the continuous spectrum is more characteristic of solids than of gases, and Mr. Slipher has adduced evidence that some nebulous matter shines by reflected light. The extreme tenuity and presumably low temperature of nebulae increase the difficulty of a purely gaseous hypothesis, which is further contradicted by the abundance of multinuclear nebulae.

I infer that it is perfectly legitimate to speculate on celestial clouds composed of matter in more than one and perhaps in all three states.

Provided that there is a limit to the expansion of gas, as G. W. Hill believed, a gaseous spheroid may assume a figure of equilibrium and this is the favorite postulate among modern cosmogonists. Some of the more regular nebulae lend color to it; but the forms of the celestial clouds are as varied as those of our atmosphere and might in large part be similarly classified. Surely some of the elongated nebulae, resembling cirrus or cirro-cumulus terrestrial clouds, must eventually develop into more highly organized forms; or, inversely, some well-developed nebulae may have originated from whisps of nebulous matter, such as abound in the sky.

It is not possible at present to assign a definite origin for these nebulous streamers. The hypothesis that the whole galaxy was nebulous at a certain epoch leads to a dilemma as was long since pointed out.[3] In 1900 Arrhenius put forward his theory that energy is degraded in the solar state but raised to a higher level in the nebular state,[4] and some such 'third law' of thermodynamics seems logically inevitable. But whether or not there is a regenerative process at work among the heavenly bodies, there are at least inequalities of action and preferential movements (so suggestively discussed by Kapteyn) which would almost inevitably lead to a carding or filamentation of nebulous masses.

It has always been impossible to suppose that nebulae were devoid of internal motion, but such movements were first demonstrated in 1914 by Messrs. Buisson, Fabry, and Bourget[5] for the Orion nebula. Their result has been confirmed by Messrs. Frost and Maney,[6] who point out that this nebula must be considered as a mass writhing and seething in irregular contortions. A similar statement must be true of all nebulae.

Consider then an elongated nebula composed of heterogeneous matter and bounded by a surface which may be very irregular but not so irregular that any part of its longest principal axis falls outside of the mass. Such a nebula would bear some resemblance to a staff or baculum which may be provided with knots, knobs, and excrescences. For the sake of brevity I will characterize such a nebula as bacular. A bacu-

lum then must be possessed of energy and moment of momentum, but I shall suppose its axis of rotation to stand at an angle to its greatest principal axis of inertia.

Such a bacular nebula would pass through some of the phases familiar to readers of cosmogony; energy would be dissipated by collisions; a part of the material would subside towards a central nucleus or several excentric nuclei; excepting for some outlying cometary detritus, the mass would tend to flatten in the invariable plane; and finally orbits of great eccentricity would or might be reduced by collisions to an approximately circular form, while of course the moment of momentum would remain nearly or quite constant. The flattening of such a nebula and its rotation about the centre of inertia of the whole system are so nearly independent that the principle of superposition is applicable; the mass may be supposed to subside into its invariable plane without flexure of the axis of the baculum, and thereafter to rotate about a line perpendicular to this axis.

The question then arises how the axis of a bacular nebula would be distorted compatibly with Kepler's third law if the orbits of the component particles were sensibly circular, or if eccentricities are neglected, only a first approximation being sought.

In its simplest form Kepler's third law expresses the equality between the attraction of a heavy point on a particle moving in a circular orbit and the centrifugal force, or normal acceleration, of the particle. If ω is the angular velocity of the particle, r its distance, and M the mass of the heavy point,

$$\frac{M}{r^2}=\omega^2 r.$$

Here M may mean the mass of a system concentrated at its centre of inertia and it is easy to see that r may be replaced by the mean distance a, of a particle moving in an elliptical orbit.

The law so expressed would be valid for the solar system, so far as two bodies are concerned, were it practicable to take the centre of inertia of that system as an origin; most astronomical problems, however, involve not absolute but relative motions, and Kepler's law must then be modified by substituting $M+m$ for $1/M$.[7] For small planets m can be neglected because the sun's centre is then almost exactly at the centre of inertia of the system.

If a system consisted of a single nucleus and a great number of minute secondary particles which, at a given epoch, were arranged on an axis,

and thereafter for a time τ described circular orbits, then, putting $\tau\omega=\varphi$ and $a=r$ the particles would be found on a curve

$$r^3\varphi^2=\text{constant}.$$

This is a spiral with a single well-defined point, which is a point of inflection. If r_0, φ_0 are the coordinates of this point it is easy to show that $\varphi_0^2=2/9$ so that

$$r^3\varphi^2=2r_0^3/9.$$

If the axial row of particles extended on both sides of the nucleus, the spiral would have two branches, each approaching the axis asymptotically from opposite sides. The length of any portion of the axis would be much extended by conversion into the spiral, the extension at any point being ds/dr, which, at the point of inflection is $\sqrt{3/2}$, while closer to the centre it is still larger.

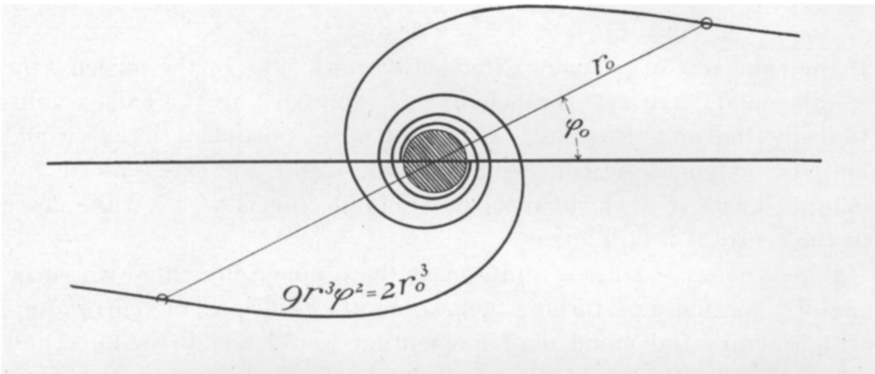
For secondary bodies of finite mass the problem presented would be one of n mutually perturbing bodies, but the principles of centre of inertia, energy, and moment of momentum would remain in force and parabolic velocities could not be attained. It would appear, therefore, that even in a system of finite masses, axially arranged, angular velocities must diminish with distance from the centre of inertia and be infinitesimal at an infinite distance. Then the axis must be distorted into a more or less irregular spiral which is asymptotic to the axis and must, therefore, have a point of inflection. These conditions would be satisfied by any curve of the form $r\varphi^{n^2}=\text{constant}$, $n^2<1$, for these spirals have a point of inflection at $\varphi_0=n\sqrt{1-n^2}$, the maximum value of which is 1/2.

Possibly perturbations might bring about approximations to some of these curves, but since perturbations are necessarily excluded from consideration, the only curve with any standing is that in which $n^2=2/3$.

Supposing, then, that the initial configuration of the system were an axial line passing through the centre of inertia and that the orbits were circular, the configuration after the lapse of a given time would vary from the spiral $r^3\varphi^2=\text{constant}$ only as a result of perturbations and this spiral must represent, to a first approximation, the axis of such spiral nebulae as have been developed from substantially rectilinear nebulous streamers, while in some other cases it will represent displacements from an initial configuration.

The diagram shows the form of the spiral excepting that near the center where it would be almost impossible to plot the curves, a nucleus is substituted. Both branches are prolonged somewhat beyond the

points of inflection. Of course, the curve in the diagram represents only the axis of the baculum. Below the diagram is shown the Whirlpool nebula[8] for comparison, and evidently a diagram could be drawn representing the mathematical spiral clothed, so to speak, with irregularly disposed nebulous material which would add to its resemblance to the photograph.



As noted above, even at the point of inflection the elongation of the original axis is in the ratio of $\sqrt{3/2}$ or 1.22, while for points nearer the origin it is more than this. Unless, therefore, the original baculum were of extraordinary uniformity, this stretching combined with local partial attractions would break up the spiral into short curved fragments. Between the point of inflection and the origin such fragments, would present their concave sides to the origin. If any such fragment were to condense to a spheroid, the centre of the spheroid would be

close to the centre of inertia of the arc corresponding to the fragment; therefore the condensed mass would be nearer to the centre of attraction than in the uncondensed state and consequently the rotation of the spheroid would be positive. Beyond the point of inflection these conditions would be reversed so that condensing short arcs of the nebulous spiral would retreat and show retrograde rotation. In systems thus developed, then, positive rotation would characterize all but the outermost portions where retrograde rotation would prevail.

A bacular nebula so short or so old that its outermost portions had completed two or three revolutions round the centre of attraction would be almost or quite indistinguishable from a circular disc, marked by furrows and closely resembling Laplace's nebula after the development of rings. From this epoch onwards all the valid conclusions drawn from Laplace's great hypothesis would be applicable to the nebula under consideration, and in particular the splendid investigations of Roche, Hill, and George Darwin on unstable orbits.

So far as I can see, a baculum of very uniform composition might develop into a star without planets, though possibly attended by a disc reflecting zodiacal light; while a very heterogeneous baculum might yield a system as complex as the Pleiades. There seems to me no reason why even the most symmetrical planetary nebulae might not have been evolved from bacula closely coiled. On the other hand, a nebula much less symmetrical than a baculum as I have defined it, can be similarly discussed. Suppose the axis of figure of an elongated nebula to be any arbitrary curve of small curvature; then the equation $r^3\varphi^2 = 2r_0^3/9$ does not represent the distorted axis but does give the displacement of points on the assumed curve. Or inversely, comparison of a well-defined spiral nebula with the spiral represented by the equation should show to what simplest form the nebula could be reduced by reversing the orbital velocities. Again, if the axis of figure of the filamentous nebula were coincident with an arc of a circle, its centre of inertia might lie outside of the nebula; and in such a case, after the extremities of the nebula had revolved about the centre of inertia a few times, the figure would be indistinguishable from an empty ring; but if a portion of the mass were arrested by collisions, the ring would show a central nucleus as does the ring nebula in Lyra.

Superficial comparison of the diagram given in this paper with photographs of the Whirlpool and other nebulae is not unfavorable to the hypothesis here developed, and it seems to me that to the first order of approximation bacular nebulae must ultimately be converted into the spiral here discussed. But the geometry of this spiral is so well characterized

that astronomical observers will probably be able to make a definite decision as to whether or not any of the existing spiral nebulae have developed from bacula or from less regular forms by substantially the same method. Till that is decided further discussion would be superfluous. It is possible that there are well developed spiral nebulae in which mutual interference has done little or nothing to reduce orbits to ellipses of small eccentricity. To such nebulae the analysis here indicated has no application, nor could their motions be formulated in the present state of science.[9]

[1] *Amer. J. Sci.*, Ser. 4, **5**, 102 (1898).

[2] Chapter VI, p. 482, of the edition of 1835.

[3] *Amer. J. Sci.*, Ser. 4, **5**, 106–107 (1898).

[4] See preface to *Worlds in the Making*, 1908.

[5] *Paris, C. R. Acad. Sci.*, **158**, 1017 (1914); and *Astrophys. J.*, **40**, 241 (1914).

[6] *Pop. Astr.*, **23**, 485 (1915).

[7] A. G. Webster, *Dynamics of Particles*, etc., 2d ed., 1912, p. 317.

[8] From a photograph taken at the Yerkes Observatory.

[9] Just in time for a reference, I have met with an interesting paper by Mr. E. J. Wilczynski, *Astrophys. J.*, **4**, 97, (1896), who pointed out that, if circular orbits are assumed, every long streak of nebulous matter must eventually be converted into a spiral as a consequence of Kepler's third law. The equation of this curve, he says, it would be easy to deduce.

A PECULIAR CLAY FROM NEAR THE CITY OF MEXICO

By E. W. Hilgard

UNIVERSITY OF CALIFORNIA

Read before the Academy, November 17, 1915. Received, November 17, 1915

In 1912 Dr. George W. Shaw, then connected with the College of Agriculture of the University of California, was requested to visit the Hacienda Santa Lucia, in the neighborhood of the City of Mexico, in order to give advice for the reclamation of certain tracts of land which were supposed to be afflicted with 'alkali,' and which had resisted the usual methods for rendering them productive. Dr. Shaw brought back samples of the soils from these alkali spots, which were as usual depressed in the middle, but found in them no excessive amounts of carbonate of soda, and that the sulphate (Glaubers salt) was mainly present in fractions of one per cent, not enough to injure vegetation.

In an attempt to leach one sample of its soluble salts he found that on pouring water on a few grams placed in a 50 cc. cylinder the substance swelled very rapidly, and over-night actually filled the cylinder to the top, making a semifluid slush. As I had never seen the like before, I undertook to investigate it.